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THEORETICAL MODELS OF BETA CEPHEI STARS CONSTRUCTED WITH NEW RADIATIVE OPACITIES

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ABSTRACT

Theoretical models of stars in the mass range of β Cephei stars have been computed with Carson's new radiative opacities from the zero-age main sequence up to the stage of core helium ignition. The new models lying off the zero-age main sequence possess cooler effective temperatures than do models based on earlier opacities, and cross the observed β Cephei strip in the H-R diagram only once. This intersection occurs during the main phase of core hydrogen burning. Nonadiabatic pulsational properties of the models during this phase of evolution have been derived in the linear approximation. The pulsation constants and period ratios for both radial ($l = 0$) and nonradial ($l = 2$) modes turn out to be systematically (though not largely) different from those possessed by models based on earlier opacities. Cases of two modes with close periods are pointed out that may possibly account for the "beat" phenomenon observed in many β Cephei stars. The observed phase lag of 90° between the light and radial-velocity curves is predicted correctly along the intersection strip for many of the modes. But all the modes that have been tested for actual instability, viz., those with $0 \leq l \leq 5$ and $0.02 \leq Q$ (day) ≤ 0.06, turn out to be stable.

Subject headings: stars: β Cephei — stars: evolution — stars: interiors — stars: pulsation

I. INTRODUCTION

Variability of the β Cephei stars has long been attributed to some sort of radial or nonradial pulsation. However, the energizing mechanism for the pulsations has so far eluded definite identification. Since the large number of these stars and the slowness of their period changes imply that the majority of them are in the main phase of core hydrogen burning (Struve 1960; Eggleton and Percy 1973; Percy 1974), it is of interest to examine whether the new radiative opacities calculated by Carson (1976) lead to any radial or nonradial pulsational instability during the main-sequence phase of evolution for stars in the mass range 9–20 $M_\odot$, which appears to be characteristic of the known β Cephei stars.

Carson's opacities, unlike earlier opacities (e.g., Cox and Stewart 1965), exhibit a "bump" due to the ultimate ionization of the CNO elements at moderate temperatures and low densities. It has recently been shown (Stothers 1976) that this bump excites radial pulsations in models of zero-age main-sequence (ZAMS) stars of very high mass, via the κ-mechanism operating in the stellar envelope. Since the bump is more prominent at lower densities, models of less massive stars that are evolving off the ZAMS should also tend to lose pulsational stability as time goes on. Nevertheless, we shall find in the present paper that this mechanism will not explain the variability of β Cephei stars, because their masses are simply too small; but a number of other possible clues to the nature of these stars will be obtained.

II. STRUCTURAL PROPERTIES

Equilibrium models for the main-sequence and early post–main-sequence phases of evolution have been constructed for stellar masses of 10.9 and 15 $M_\odot$, with two different initial (hydrogen, metals) abundances, $(X, Z) = (0.73, 0.02)$ and $(X, Z) = (0.71, 0.04)$. For convenience in the later pulsation calculations, some slight simplifications have been introduced in the energy generation rate and in the gradient of mean molecular weight left behind by the retreating convective core. Convection, which occurs not only in the core but also in the helium and CNO ionization zones of the envelope, has been treated by the standard mixing-length theory, with the ratio of mixing length to pressure scale height taken to be $\alpha = 1$. Since in the stellar models derived here core convection is adiabatic and the envelope convection zones are rather thin, our particular choice of α turns out to be unimportant for the structure. Rotation of the star is neglected in the present work. Selected models from the new evolutionary sequences are listed in Table 1, where τ represents the time elapsed from the ZAMS and β_c is the ratio of gas pressure to total pressure at the center of the star.

Evolutionary tracks on the H-R diagram are shown for the new models in Figure 1. Observational data for proven β Cephei stars are also plotted, having been adapted from the work of Balona and Feast (1975), with the omission of those stars considered by Shaw (1975) to be only *suspected* members of this class. It would appear that the β Cephei stars, particularly those

TABLE 1

EQUILIBRIUM AND RADIAL PULSATIONAL PROPERTIES OF SELECTED MODELS

PARAMETER	10.9 $M_\odot$ $(X, Z) = (0.71, 0.04)$					10.9 $M_\odot$ $(X, Z) = (0.73, 0.02)$					15 $M_\odot$ $(X, Z) = (0.73, 0.02)$				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
X_c	0.710	0.528	0.335	0.138	0.029	0.730	0.522	0.306	0.129	0.064	0.730	0.526	0.307	0.127	0.064
$\log L/L_\odot$	3.806	3.916	4.006	4.066	4.080	3.826	3.936	4.026	4.076	4.090	4.244	4.360	4.460	4.514	4.530
$\log T_e$	4.373	4.356	4.322	4.271	4.250	4.404	4.385	4.352	4.313	4.302	4.453	4.432	4.388	4.337	4.320
β_c	0.931	0.913	0.890	0.862	0.844	0.934	0.914	0.888	0.863	0.852	0.896	0.867	0.828	0.792	0.778
$\log \rho_c/\langle\rho\rangle$	1.714	1.991	2.365	2.845	3.131	1.621	1.904	2.280	2.674	2.826	1.631	1.947	2.409	2.893	3.092
$\tau(10^7$ yr)	0.00	0.81	1.35	1.67	1.76	0.00	0.88	1.42	1.69	1.75	0.00	0.55	0.91	1.08	1.12
ω_0^2	11.8	12.1	11.1	9.94	9.52	10.6	11.8	11.8	11.2	11.0	10.1	12.0	10.9	9.58	9.18
ω_1^2	16.2	18.5	19.4	20.1	20.1	16.1	17.7	18.9	19.3	19.3	15.3	16.9	18.6	19.5	19.6
ω_2^2	25.6	26.5	29.4	30.5	30.3	26.6	26.6	27.5	29.3	29.7	25.2	25.0	28.1	29.1	28.7
Π_0(hr)	2.60	3.49	5.41	8.96	10.9	2.30	2.98	4.39	6.45	7.21	2.92	3.80	6.41	10.7	12.7
Π_1(hr)	2.22	2.82	4.09	6.30	7.48	1.87	2.44	3.47	4.92	5.43	2.38	3.20	4.92	7.51	8.68
Π_2(hr)	1.77	2.36	3.32	5.12	6.10	1.45	1.99	2.88	3.98	4.38	1.85	2.63	4.00	6.14	7.17
$1/K_0$*(yr)	6.33	2.02	1.98	1.76	1.81	23.9	4.36	2.27	2.24	2.18	13.2	0.495	0.268	0.334	0.348
$1/K_1$*(yr)	1.05	0.201	0.175	0.070	0.049	2.03	0.636	0.267	0.159	0.133	0.367	0.139	0.094	0.048	0.032
$1/K_2$*(yr)	0.100	0.083	0.033	0.009	0.007	0.099	0.067	0.055	0.026	0.019	0.047	0.038	0.025	0.006	0.005
Π_1/Π_0	0.854	0.809	0.756	0.704	0.688	0.812	0.817	0.792	0.762	0.753	0.813	0.842	0.768	0.701	0.684
Π_2/Π_0	0.679	0.676	0.613	0.571	0.561	0.631	0.667	0.655	0.617	0.607	0.634	0.694	0.624	0.574	0.565

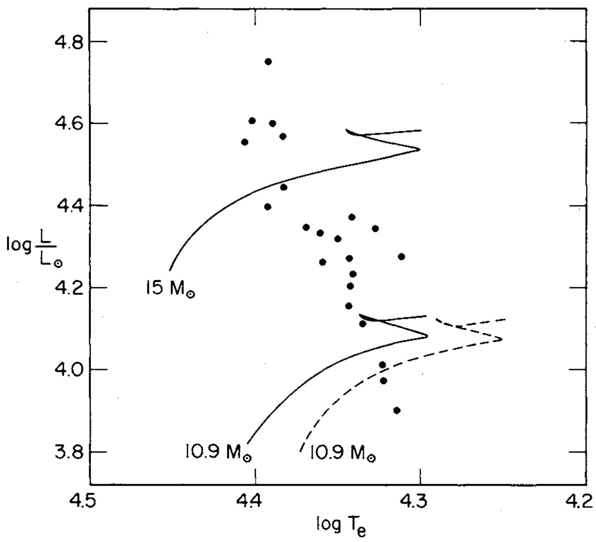

FIG. 1.—Evolutionary tracks in the theoretical H-R diagram for stars of 10.9 and 15 $M_\odot$ evolving from the ZAMS to the onset of core helium burning. Solid lines refer to a metals abundance of $Z = 0.02$; dashed line, to $Z = 0.04$. Dots represent known β Cephei stars.

at high masses, lie too close to the ZAMS to be evolving either in the phase of secondary contraction or in a subsequent phase, if Carson's opacities are correct. On the other hand, the somewhat smaller opacities of Cox and Stewart yield higher effective temperatures for equivalently evolved stellar models, with the consequence that the inferred evolutionary phase of β Cephei stars could, for those opacities, be late core hydrogen burning, secondary contraction, or envelope reexpansion (see, e.g., Lesh and Aizenman 1973). But it is relevant to recall the observational evidence referred to in § I in favor of the main phase of core hydrogen burning for β Cephei stars, and to note that stellar models constructed with the Cox-Stewart opacities *may* require an excessively large initial hydrogen or metals abundance to match the observations in the H-R diagram for this phase of evolution (Lesh and Aizenman 1973; Jensen 1974; Jones and Shobbrook 1974; Balona and Feast 1975). A similar, though more extreme, difficulty was found earlier for stellar models constructed with a purely electron-scattering opacity (Stothers 1965), and indicated the need for a larger opacity.

Odell (1974) has shown that no realistic models constructed with the Cox-Stewart opacities are able to reproduce the observed apsidal-motion constant of α Vir, which is a β Cephei star of $10.9 \pm 1.3\ M_\odot$ in a binary system (Herbison-Evans *et al.* 1971; Shobbrook, Lamb, and Herbison-Evans 1972). However, the use of Carson's opacities resolves this problem and suggests that α Vir has finished about 80 percent of its expected lifetime for core hydrogen burning (Stothers 1974).

III. PULSATIONAL PROPERTIES

Radial and nonradial pulsations of the new stellar models have been studied by applying linear nonadiabatic perturbations to the models. The calculation of the radial pulsations is exact, following our earlier procedure (Stothers 1976). For nonradial pulsations, the following approximations have been made: (1) the Eulerian perturbation of the gravitational potential is ignored; and (2) horizontal energy exchanges are neglected. The first approximation does not lead to any sizable error in the pulsational eigenfrequencies and eigenfunctions (e.g., Wan Fook Sun and Van der Borght 1966; Robe 1968), especially when the effective polytropic index of the star is high; and the second approximation is found by supplemental calculations to be quite good, except for very low pulsation eigenfrequencies and very high degrees of the spherical harmonic. These two simplifying approximations allow the nonradial pulsation equations to be written as slightly modified versions of the equations for radial pulsations. Thus, for example, transformation of Zahn's (1975) nonradial pulsation equations into a form employing solely Lagrangian perturbations yields a set of equations similar to the radial pulsation equations of Baker and Kippenhahn (1965), but with the addition of several extra terms. The work integral (or running stability integral) is identical, however, because of our second approximation above; but the integral for the kinetic energy of pulsation contains an extra term to take account of the horizontal motion of the layers (e.g., Dziembowski 1971). In a chemically inhomogeneous region of the star, the gradient of mean molecular weight does not appear explicitly in the pulsation equations if Lagrangian perturbations are adopted, and therefore is treated exactly here. The convective flux is assumed in the present work not to interact at all with the pulsations. The following notation is adopted: l, the degree of the spherical harmonic $Y_l^m(\theta, \phi)$ in which the Eulerian perturbations of the physical variables are expanded; $\omega^2 = (2\pi/\Pi)^2 R^3/GM$, the square of the nondimensional pulsational eigenfrequency, where Π is the period of pulsation; $Q = \Pi(M/M_\odot)^{1/2}(R/R_\odot)^{-3/2} = 0.116/\omega$, the pulsational Q-value in days; and K^*, the stability coefficient, whose reciprocal is the e-folding time for the growth or decay of the pulsational amplitudes (the asterisk indicates our neglect of running waves in the atmosphere).

a) *Radial Pulsations*

Characteristics of the three lowest modes of radial pulsation are given in Table 1 for a few selected stellar models. By comparing these results with earlier results based on the rather small Cox-Stewart opacities (Davey 1973; Lesh and Aizenman 1974; Osaki 1975) and on the limiting case of pure electron-scattering opacity (Stothers 1965), we find that there emerges a moderate dependence of the various Q-values and period ratios on the contribution of atomic absorption to the envelope opacity (the core opacity in all cases is essentially electron scattering). This dependence is due to the corresponding change of the effective polytropic index of the envelope. A similar behavior of the Q-values and period ratios occurs in the case of pure polytropes of increasing index n (e.g., Ledoux and Walraven 1958).

At the intersection between the new evolutionary tracks and the observed β Cephei strip on the H-R diagram, the theoretical pulsation constants are found to be virtually independent of the stellar mass and initial chemical composition, viz., $Q_0 = 0.035$, $Q_1 = 0.027$, and $Q_2 = 0.022$. The most recent empirical values of the pulsation constants for β Cephei stars are $\langle Q \rangle = 0.030$ (Jones and Shobbrook 1974), 0.025 (Lesh and Aizenman 1974), and 0.027 (Balona and Feast 1975); therefore, the weight of present evidence concerning Q seems to favor a first-overtone instability if the pulsations are, in fact, radial. The multiple periods claimed to exist in β Cephei stars by van Hoof (1962) require reidentification before they can be used in comparison with the present models.

All the radial modes examined in the present models, just like those in earlier models based on different opacities, turn out to be stable. The margin of stability, however, as measured by the run of the stability integral through the star rather than by the e-folding time $1/K^*$, diminishes in the present models as the envelope expands during evolution. The contribution to the total stability integral from nuclear reactions in the stellar core is comparatively slight, since the radial pulsation amplitudes are very small near the center.

b) *Nonradial Pulsations*

Quadrupole ($l = 2$) pulsations have also been computed for the stellar models of Table 1. The results for the lowest modes are presented for a few of the stellar models in Table 2. Identification of the modes has been based on Osaki's (1975) prescription, the notation being Cowling's (1941). A small error in the eigenvalues has been incurred by our neglect of the Eulerian perturbation of the gravitational potential; however, this error is estimated to be of the same sign for all eigenvalues and of approximately the same size for neighboring eigenvalues. Consequently, the differences and the ratios of neighboring eigenvalues are expected to be fairly accurately determined. By examining earlier work, we find that the derived Q-values and period ratios exhibit the same modest dependence on the contribution of atomic absorption to the envelope opacity as was the case for the radial modes (Cox-Stewart opacities: Baker and Dziembowski 1969, Harper and Rose 1970, Deupree 1974, Osaki 1975; electron-scattering opacity: Smeyers 1963, 1967, Van der Borght and Wan Fook Sun 1975; and simple polytropes: Robe 1968). In the evolved models, the spectrum of nonradial quadrupole modes is denser than in previously published models, because the central condensation of the new models is higher. But in conformity with earlier work, most of the spatial oscillations of the radial eigenfunctions are confined to the zone with a gradient of mean molecular weight, and no nodes appear in the convective core or close to the stellar surface.

The two lowest p modes in those stellar models that lie nearest the observed β Cephei strip in the H-R diagram have periods that are fairly close to one another. (A similar convergence of the periods of p modes occurs in the partially evolved stellar models based on Cox-Stewart opacities, but the relationship of these models to the observed β Cephei strip is unclear.) This closeness of periods may possibly be responsible for the "beat" phenomenon that is observed in the light curves of many β Cephei stars (e.g., Struve 1955) if these two modes happen to be excited. The theoretical period difference has almost the required smallness, although the periods themselves seem to be somewhat too short ($Q \approx 0.020$). Another possibility, which has been discussed before in the literature, is that coupling may occur between a radial mode and a nonradial mode of nearly the same frequency (e.g., between the fundamental radial mode and the nonradial f mode, according to Chandrasekhar and Lebovitz 1962; Hurley, Roberts, and Wright 1966; Harper and Rose 1970; Deupree 1974). The present evolved models reveal the existence of several possible resonances between individual radial and nonradial modes; for example, a near resonance seems to occur between the first radial overtone and the nonradial f mode in those models that lie nearest the observed β Cephei strip. However, it should be cautioned that the nonradial periods have not been calculated exactly, and therefore firm conclusions about the apparent resonances cannot be drawn.

Unfortunately, as is the case for the radial modes, all the nonradial modes studied here are found to be stable, although the margin of stability does decrease with advancing evolution. A survey of the lowest nonradial modes for $l = 1$, 3, 4, and 5 has likewise uncovered no instability in the stellar models of Table 2. Pulsational damping below the main driving layers in the CNO ionization zone turns out to be too strong. Since the nonradial modes possess vanishing temperature fluctuations at the stellar center, nuclear energy release is also an ineffective driving mechanism for such modes in the present stellar models.

IV. DISCUSSION

If Carson's opacities are reasonably good approximations of the "true" opacities, then models of stars that have evolved off the ZAMS may possess cooler effective temperatures than has been previously believed. As a consequence, β Cephei stars could be tentatively inferred to be in the main phase of core hydrogen burning. Although all the radial and nonradial pulsation modes that have been studied in the present models prove to be quite stable, the closeness of the periods of the two lowest nonradial p modes of $l = 2$ (or, alternatively, a resonance between an individual radial and nonradial mode) may possibly be responsible for the "beat" phenomenon observed in many β Cephei stars if these modes are somehow excited.

Several speculations will be briefly discussed to explain the instability of these stars. First, convective

TABLE 2

NONRADIAL PULSATIONAL ($l = 2$) PROPERTIES OF SELECTED MODELS

	10.9 $M_\odot$ $(X, Z) = (0.71, 0.04)$						10.9 $M_\odot$ $(X, Z) = (0.73, 0.02)$						15 $M_\odot$ $(X, Z) = (0.73, 0.02)$								
	Model 1			Model 3			Model 1			Model 5			Model 1			Model 3			Model 5		
MODE	ω^2	Π(hr)	$1/K^*$(yr)	ω^2	Π(hr)	$1/K^*$(yr)	ω^2	Π(hr)	$1/K^*$(yr)	ω^2	Π(hr)	$1/K^*$(yr)	ω^2	Π(hr)	$1/K^*$(yr)	ω^2	Π(hr)	$1/K^*$(yr)	ω^2	Π(hr)	$1/K^*$(yr)
g_7^+...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	5.72	16.1	75.9
g_6^+...	...	...	...	...	...	...	...	...	...	4.80	10.9	1400	...	...	...	...	...	...	7.31	14.2	70.7
g_5^+...	...	...	...	...	...	...	...	...	...	7.43	8.76	309	...	...	...	...	...	...	9.42	12.5	0.433
g_4^+...	...	...	...	...	...	...	...	...	...	9.19	7.88	15.0	...	...	...	...	...	...	11.3	11.4	1.79
g_3^+...	...	...	...	5.49	7.68	1360	...	...	...	11.6	7.03	0.710	...	...	...	5.24	9.26	318	13.3	10.5	2.05
g_2^+...	1.50	7.30	6910:	8.17	6.29	54.8	1.16	6.95	18400:	15.3	6.11	7.55	1.05	9.10	3850:	7.27	7.86	138	20.4	8.51	0.029
g_1^+...	3.36	4.88	6080:	11.4	5.33	0.567	2.62	4.62	12500:	19.7	5.38	0.114	2.33	6.09	10400:	11.2	6.34	0.257	23.8	7.88	0.292
f.....	12.5	2.53	2.25	19.6	4.07	0.166	11.6	2.20	7.97	29.7	4.38	0.021	11.8	2.71	3.01	18.8	4.89	0.096	29.3	7.09	0.005
p_1....	17.7	2.13	0.381	29.2	3.33	0.036	17.4	1.79	0.626	39.0	3.82	0.010	17.1	2.25	0.253	28.2	3.99	0.025	38.3	6.21	0.003
p_2....	26.6	1.73	0.093	39.7	2.86	0.009	27.5	1.43	0.091	42.0	3.68	0.007	26.3	1.82	0.043	38.4	3.42	0.005	50.6	5.40	0.007
p_3....	39.8	1.42	0.029	46.7	2.63	0.010	40.1	1.18	0.026	...	...	...	38.8	1.49	0.015	46.3	3.11	0.005	...	...	...
p_4....	55.6	1.20	0.008	...	...	...	56.6	0.99	0.008	...	...	...	54.6	1.26	0.005	...	...	...	...	...	...

instability in the CNO ionization zone may lead to driving of radial or nonradial pulsations of the whole star. Since such a convection zone exists only for stellar higher than $\sim 10\ M_\odot$, the lower mass limit of known β Cephei stars could possibly be explained. However, the CNO convection zone probably is too small and too close to the surface to be effective in this regard. Also, formal semiconvective instability (based on either the Schwarzschild or the Ledoux criterion) in the layers with a gradient of mean molecular weight fails to occur before central hydrogen exhaustion in any of the stellar models for masses less than $\sim 14\ M_\odot$, and therefore does not seem to be a serious candidate as a mechanism for the observed instability. Another possibility to keep in mind is that future improvements in stellar opacity calculations may lead to a larger CNO bump (or other bumps) in the intermediate temperature range. This would enhance the effect of the κ-mechanism in stars of suitable mass. But an upper mass limit for pulsational instability could be imposed by the growing importance of convection, as the stellar mass is raised, in layers where the opacity bump is largest (since the analogous growth of convection in the hydrogen and helium ionization zones of classical Cepheids, as the effective temperature is lowered, may be the reason for the attainment of pulsational stability by these stars at low effective temperatures, according to Baker and Kippenhahn 1965).

Finally, it is worth mentioning that *only* where the theoretical evolutionary tracks intersect the observed instability strip on the H-R diagram do the new models predict a phase lag of about 90° (as is actually observed) between the light and radial-velocity curves of β Cephei stars, for both radial modes and nonradial quadrupole modes with $Q = 0.02$–0.04 day. This coincidence is interesting because strong departures from the usual adiabatic phase relation arise in the new models near the top of the CNO ionization zone owing to the rapid change of slope of the opacity curve there; however, the phase lag returns at the surface to the adiabatic value of 90° (maximum light at minimum radius) when $X_c \approx 0.3$. This fact, plus various other features of the models based on Carson's opacities, removes our previous objection (Stothers and Simon 1969) to the suggestion of Underhill (1966) that the ultimate ionization of the CNO elements *might* be the main factor responsible for maintaining pulsational instability in β Cephei stars. It remains to be seen, however, what the instability mechanism actually is.

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